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Robust/Optimal Temperature Profile Control Using Neural Networks

Vivek Yadav[1], Radhakant Padhi[2] and S. N. Balakrishnan[3]

Abstract— An approximate dynamic programming (ADP) based neurocontroller is developed for a heat transfer application. Heat transfer problem for a fin in a car's electronic module is modeled as a nonlinear distributed parameter (infinite-dimensional) system by taking into account heat loss and generation due to conduction, convection and radiation. A low-order, finite-dimensional lumped parameter model for this problem is obtained by using Galerkin projection and basis functions designed through the 'Proper Orthogonal Decomposition' technique (POD) and the 'snap-shot' solutions. A suboptimal neurocontroller is obtained with a single-network-adaptive-critic (SNAC). Further contribution of this paper is to develop an online robust controller to account for unmodeled dynamics and parametric uncertainties. A weight update rule is presented that guarantees boundedness of the weights and eliminates the need for persistence of excitation (PE) condition to be satisfied. Since, the ADP and neural network based controllers are of fairly general structure, they appear to have the potential to be controller synthesis tools for nonlinear distributed parameter systems especially where it is difficult to obtain an accurate model.

1. INTRODUCTION

Most of the real-world engineering problems are distributed in nature and can be described by a set of partial differential equations (PDEs) (e.g. heat transfer, fluid flow, flexible structures etc.) for which one must take the spatial distribution into account. The analysis and controller design for such systems are often far more complex than for the lumped parameter systems (which are governed by a finite number of ordinary differential equations).

There are different theoretical methods for the control of distributed parameter systems in the infinite dimensional operator theory framework however most of these methods are confined to linear systems and are difficult to implement. One approach is to get a low-order lumped parameter model of the system by using a set of orthogonal basis functions via Galerkin projection [Holmes]. This method results in a lumped parameter system that represents the properties of the original system. The choice of basis functions is very important in this method. If arbitrary orthogonal functions (e.g. Legendre polynomials, Chebyshev polynomials, Fourier functions etc.) are chosen, it cannot be guaranteed that most of the energy is captured. For this reason, a lot of attention is being given to the technique of POD [Holmes, [a]Ravindran]. "Sanp-shot" solutions are used to design problem based orthogonal functions. It has been proved that this method leads to optimal representation of PDE systems with the least number of basis functions [Holmes, [a]Ravindran]. It is important to have a 'good' controller for simulating the system to collect the snap-shot solutions for POD technique. Quite often an open loop control is used for this purpose. A drawback of this scheme is that it may not excite some modes that are excited in a feedback situation. This issue is addressed by using a feedback linearized controller to generate the snap-shot solutions [Slotine]. Since this controller, though non-optimal is stabilizing, the POD modes capture the snap-shots that are most likely to be encountered with use of optimal control.

Almost all of the real-life problems can be formulated in framework of optimal control. The SNAC technique can be used to solve two point boundary value [Bryson] problem arising in optimal control. As only one network is used in SNAC when compared to other adaptive critic designs, the training process takes lesser time. *The main contribution of this paper is to offer a robust controller design for* ***distributed parameter*** *systems by using an online neural network to compensate for unmodeled dynamics and parametric uncertainties.* The key idea is to capture the unmodeled dynamics using a neural network, the weights of which are updated online. But the gradient based weight update scheme cannot guarantee bounded weight estimates when approximating non-linear functions. *To ensure that*

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weights are bounded, the PE condition needs to be satisfied. A weight update scheme is proposed which guarantees boundedness of weights.

The rest of the paper is organized as follows. Section 2 presents the heat transfer problem considered. Section 3 presents a discussion on POD technique. Brief discussions on SNAC and online neural networks are provided in sections 4 and 5 respectively. In section 6 shows how the schemes are applied to the heat transfer problem. Simulation results are presented in section 7. Concluding remarks are provided in the final section.

2. MATHEMATICAL MODEL FOR THE PROBLEM

2.1 *Mathematical Model*

Mathematical model of heat transfer in car's electronic part is developed in this section by using basic thermal physics [Miller]. The development is illustrated in Figure 1.

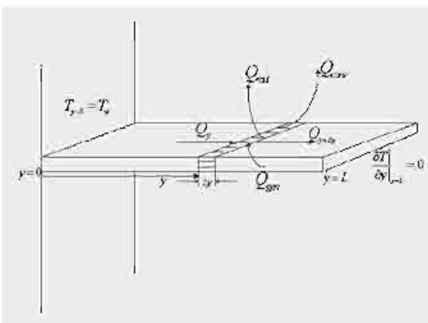

Figure 1. Pictorial Representation of the Problem

The The law of conservation of energy in the infinitesimal volume at a distance y, having length Δy (as depicted in Figure 1), for this problem is described in (1)

$$Q_y + Q_{gen} = Q_{y+\Delta y} + Q_{conv} + Q_{rad} + Q_{chg} \tag{1}$$

where Q_y is the entering rate of heat conduction, Q_{gen} is the rate of heat generation, $Q_{y+\Delta y}$ is the exiting rate of heat conduction, Q_{conv} is the rate of heat convection, Q_{rad} is the rate of heat radiation, and Q_{chg} is the rate of heat change. Substituting for the expressions for the terms in (1) and normalizing using, $X = \dfrac{T - T_{\infty_2}}{T_{\max} - T_{\infty_2}}$ where $T_{\max}$ is the maximum operating temperature ($=300°C$) and $\tau = \dfrac{t}{F}$ where F (=500) is introduced to reduce the time scale and also defining $\alpha_1 = (k/\rho C)$, $\alpha_2 = -(Ph)/(A\rho C)$, $\alpha_3 = -(P\varepsilon\sigma)/(A\rho C)$ and $\beta = 1/(\rho C)$, the dynamics of error $x(t,y) = X(y) - X_d(t,y)$ can be written as,

$$\frac{\partial x}{\partial \tau} = A_0\left(\frac{\partial^2 x}{\partial y^2}\right) + A_1(X_d)x + A_2(X_d)x^2 + A_3(X_d)x^3 + A_4(X_d)x^4$$
$$+\left[\alpha_1\left(\frac{\partial^2 X_d}{\partial y^2}\right) + \alpha_2\left(\Delta T X_d + T_{\infty_2} - T_{\infty_1}\right) + \alpha_3\left((\Delta T X_d + T_{\infty_2})^4 - T_{\infty_1}^4\right)\right] + S_1 \tag{2}$$

where,

$$\begin{aligned} A_0(X_d) &= \alpha_1 F & A_3(X_d) &= 4\alpha_3 \Delta T^2 F\left(\Delta T X_d + T_{\infty_2}\right) \\ A_1(X_d) &= F\left(\alpha_2 + 4\alpha_3\left(\Delta T X_d + T_{\infty_2}\right)^3\right) & A_4(X_d) &= \alpha_3 F \Delta T^3 \\ A_2(X_d) &= 6\alpha_3 \Delta T F\left(\Delta T X_d + T_{\infty_2}\right)^2 & \Delta T &= T_{\max} - T_{\infty_2} \end{aligned} \tag{3}$$

The boundary condition in terms of normalized variables is,

$$\left.\frac{\partial x}{\partial y}\right|_{y=0} = \frac{c}{\Delta T} - \left.\frac{\partial X_d}{\partial y}\right|_{y=0}, \quad \left.\frac{\partial x}{\partial y}\right|_{y=L} = -\left.\frac{\partial X_d}{\partial y}\right|_{y=L} \tag{4}$$

where the value of c will be dictated by the temperature profile $T(t,y)$ at $y=0$. The meanings of various parameters and the numerical values used in this research are given in Table 1.

Table 1: Parameter Definitions and Numerical Values

Parameter	Definition	Value
k	Thermal conductivity	$180\ W/(m°C)$
A	Cross sectional area	$2\ cm^2$
P	Perimeter	$9\ cm$
h	Convective heat transfer coefficient	$5\ W/(m°C)$
T_{∞_1}	Temperature of the medium in immediate surrounding	$30°C$
T_{∞_2}	Temperature at a far place in direction normal to surface	$-40°C$
$\in$	Emissivity of the material	.2
σ	Stefan-Boltzmann constant	$5.669\times10^{-8}\ W/m^2$
ρ	Density of the material	$2700 kg/m^3$
C	Specific heat of the material	$0.86 kJ/(kg°C)$

2.2 *Controller Objectives*

The main objective of the controller is to make the system reach the desired temperature profile on the fin, $T(y) \to T_d(y)$ as time $t \to \infty$, where $T_d(y)$ is a desired temperature distribution along the fin. The steady state control solution S_1^*, is obtained by substituting $x=0$ in (2) and imposing the steady-state condition.

$$S_1^*(y) = -\frac{1}{\beta}\left[\alpha_1 \Delta T\left(\frac{\partial^2 X_d}{\partial y^2}\right) + \alpha_2\left(\Delta T X_d + T_{\infty_2} - T_{\infty_1}\right) + \alpha_3\left(\left(\Delta T X_d + T_{\infty_2}\right)^4 - T_{\infty_1}^4\right)\right] \tag{5}$$

The S_1^* in (5) acts as a feedforward controller for this problem. A feedback controller is added to this control to form the total controller.

2.3 *Feedback Controller*

The actual temperature and control variables are split, as shown by (6), due to the availability of desired final (steady) state temperature profile $X_d(y)$ and associated feedforward controller $S_1^*(y)$.

$$S_1(t,y) = S_1^*(y) + u(t,y) \tag{6}$$

In (6), $u(t,y)$ is the deviation in non-dimensional control from its respective steady state value. Thus (2) becomes

$$\frac{\partial x}{\partial t} = \alpha_1\left(\frac{\partial^2 x}{\partial y^2}\right) + f(x, X_d) + u \tag{7}$$

where $f(x, X_d) \triangleq \left(A_1(X_d)x + A_2(X_d)x^2 + A_3(X_d)x^3 + A_4(X_d)x^4\right)$

The purpose of the feedback controller $u(t,y)$ is to eliminate the deviations from the steady state conditions, i.e. $x(t,y) \to 0$ as time $t \to \infty$. This is achieved by minimizing a cost function,

$$J = \frac{1}{2}\int_0^\infty \int_0^L \left(q\,x^2 + r\,u^2\right) dy\, dt \tag{8}$$

In (8) $q \geq 0$ and $r > 0$ are the weights that express the designer's concern for excessive deviations from the nominal and the control effort respectively.(7)–(8) define the complete optimal control problem.

I. 3. Proper Orthogonal Decomposition: A Brief Review

Proper Orthogonal Decomposition (POD) when used in a Galerkin procedure is a technique for determining an optimal set of basis functions, which results in a low order finite dimensional model for an infinite-dimensional system. The first step in this process is to generate a set of "snapshot" solutions. The snapshots are time-histories of solutions of the DPS generated by starting the solution process from different initial conditions that satisfy the boundary conditions (or data from experimentation). Details on getting optimal basis functions from the snapshots can be found in [Holmes]. For the problem considered, 5 basis functions were found to give satisfactory results.

3.1 *Lumped Parameter Problem*

The reduction of the infinite-dimensional problem to a finite set of ordinary differential equations and the related cost function are explained in this subsection. After obtaining the basis functions $\varphi(y)$, the states and the control $x(t,y)$ and $u(t,y)$ respectively are expressed as follows.

$$x(t,y) = \sum_{j=1}^{\tilde{N}} \hat{x}_j(t)\,\varphi_j(y)$$
$$u(t,y) = \sum_{j=1}^{\tilde{N}} \hat{u}_j(t)\,\varphi_j(y) \tag{9}$$

One can notice that both $x(t,y)$ and $u(t,y)$ have the same basis functions. The rationale for this is that a *state feedback* controller spans a subspace of the state variables and hence, the basis functions for the state are assumed to be capable of spanning the controller as well. Substituting these expansions of state and controller variables in (7) and taking the Galerkin projection on the basis function φ_i (i.e. taking the inner product with respect to φ_i for all φ_i functions), and using the fact that the basis functions are orthonormal , we get

$$\dot{\hat{X}} = A\hat{X} + \hat{f}\left(\hat{X}\right) + B\hat{U} \tag{10}$$

where $\hat{X} = \left[\hat{x}_1, \ldots, \hat{x}_{\tilde{N}}\right]^T$, $\hat{U} = \left[\hat{u}_1, \ldots, \hat{u}_{\tilde{N}}\right]^T$ .Other terms are defined as follows

$$A = \alpha_1\left[a_{ij}\right] + \alpha_2 I_{\tilde{N}}$$
$$a_{ij} = \left\langle \varphi_i, \varphi_j''\right\rangle = \int_0^L \varphi_i\,\varphi_j''\,dy = \left[\varphi_i\,\varphi_j'\right]\Big|_{y=0}^{y=L} - \int_0^L \varphi_i'\,\varphi_j'\,dy \tag{11}$$
$$\hat{f}_i\left(\hat{X}\right) = \alpha_3\left\langle f(x), \varphi_i\right\rangle = \int_0^L f(x)\,\varphi_i\,dy$$
$$B = \beta I_{\tilde{N}}$$

Next, in the cost function (8),

$$\int_0^L q x^2 dy = q\langle x, x\rangle = q\left\langle\left(\sum_{i=1}^{\tilde{N}} \hat{x}_i\,\Phi_i\right), \left(\sum_{j=1}^{\tilde{N}} \hat{x}_j\,\Phi_j\right)\right\rangle$$
$$= q\sum_{j=1}^{\tilde{N}} \hat{x}_j\,\hat{x}_j = \hat{X}^T Q \hat{X} \tag{12}$$

where $Q = q\,I_{\tilde{N}}$. Similarly $\int_0^L r u^2 dy = \hat{U}^T R \hat{U}$ (13)

where $R = r\,I_{\tilde{N}}$. Thus the cost function (8),can be written as

$$J = \frac{1}{2}\int_0^\infty \left(\hat{X}^T Q \hat{X} + \hat{U}^T R \hat{U}\right) dt \tag{14}$$

4. Approximate Dynamic Programming

In this section, discussion on optimal control of distributed parameter systems is presented in an ADP framework. Development in this section will subsequently be used in the synthesis of an optimal neurocontroller for the cooling fin in automobiles.

4.1 *Problem Description and Optimality Conditions*

Consider a system given by,

$$\hat{X}_{k+1} = F(\hat{X}_k) + \hat{B}\hat{U}_k \tag{15}$$

where $\hat{X}_k$ and $\hat{U}_k$ represent the $n \times 1$ state vector and $m \times 1$ control vector respectively at time step k. The objective is to find a control $\hat{U}_k$ that minimizes the scalar cost function

$$J = \sum_{k=1}^{N-1} \Psi_k\left(\hat{X}_k, \hat{U}_k\right) \tag{16}$$

where N represents the number of discrete time steps. Note that when N is large, (16) represents the cost function for an infinite horizon problem. The conditions for optimality are,

$$\hat{U}_k = -\hat{R}^{-1}\hat{B}^T \lambda_{k+1} \tag{17}$$

The co-state propagation equation is.

$$\lambda_k = \left(\frac{\partial \Psi_k}{\partial \hat{X}_k}\right) + \left(\frac{\partial \hat{X}_{k+1}}{\partial \hat{X}_k}\right)^T \lambda_{k+1} = G(\hat{X}_k, \lambda_{k+1}, \hat{U}_k) \quad (18)$$

By using the relationships in (15),(17) , (18) and, the neural network based control can be synthesized as discussed below

4.2 *Single Network Adaptive Critic(SNAC)*

Typically, ADP based problems are solved by using two networks in a dual heuristic programming formulation (DHP): one to capture the relationship between the states and the control at stage k and the second to capture the relationship between the states and the costates at stage k. In contrast, the SNAC captures the relationship between the states at k and the costates at (k+1). SNAC can be used in problems which are control affine and therefore, the optimal control satisfies the relation (17). Details on training of SNAC can be found in [Padhi]

5. ROBUST CONTROL DESIGN

In this section, a robust control design technique is presented which augments the optimal control to compensate for unmodeled dynamics. The scheme is developed for the lumped parameter model and the control thus obtained is converted through basis functions to a spatially continuous control which is applied to the distributed parameter system.

5.1 *Problem formulation*

Consider a nominal nonlinear system model given by,

$$\frac{\partial x(t,y)}{\partial t} = f\left(x(t,y), \frac{\partial x(t,y)}{\partial y}, \frac{\partial^2 x(t,y)}{\partial^2 y}, \cdots\right) + \beta u(t,y) \quad (19)$$

let the true model with unmodeled dynamics be given by

$$\frac{\partial x(t,y)}{\partial t} = f\left(x(t,y), \frac{\partial x(t,y)}{\partial y}, \frac{\partial^2 x(t,y)}{\partial^2 y}, \cdots\right) \quad (20)$$
$$+\beta u(t,y) + D\left(x(t,y), y\right)$$

where $D\left(x(t,y), y\right)$ represents the bounded uncertainty not captured by the nominal model.

The goal is to find an extra control that can offset the effects of this uncertainty and help perform close to nominal system behavior.

5.2 *Reduced Order Lumped Parameter System*

Note that the uncertainty can be expanded as follows.

$$D\left(x(t,y), y\right) = \sum_{i=1}^{\tilde{N}} \hat{d}_i(t)\varphi_i(y) \quad (21)$$

By using (21) and taking inner product with the basis functions, a reduced order model for the true plant is obtained as follows,

$$\dot{\hat{X}} = \hat{F}(\hat{X}) + B\hat{U} + \hat{D}(\hat{X}) \quad (22)$$

where

$$\hat{F}_i(\hat{X}) = \left\langle f\left(x(t,y), \frac{\partial x(t,y)}{\partial y}, \frac{\partial^2 x(t,y)}{\partial^2 y}, \cdots\right), \varphi_i\right\rangle$$
$$B = \beta I_{\tilde{N}} \quad (23)$$
$$\hat{D}_i(\hat{X}) = \left\langle D(x(t,y),y), \varphi_i\right\rangle = \hat{d}_i(t)$$

5.3 *Uncertainty Modeling with Online Network*

This section presents the development of online neural network for compensation of uncertainty. The output of the neural network is obtained by passing the input through a set of basis functions and multiplying with a weight matrix that is updated online as the system proceeds. The output of the network can be written as $W'\phi(\hat{X})$ where W is the matrix of weights and $\phi(\hat{X})$ is a vector of basis functions. The basis functions are chosen in such a way that $1 \le \left|\phi(\hat{X})\right| \le M$. It is known that within a compact set of state $\hat{X}$ there exists a matrix of weights and a vector of basis functions that can approximate the uncertainty ($\hat{D}(\hat{X})$) in within any desired accuracy i.e.

$$\hat{D}(\hat{X}) = W_{ideal}'\phi(\hat{X}) + \varepsilon \quad (24)$$

where W_{ideal} is the matrix of ideal weight and ε is the approximation error of the neural network and for any positive number $\varepsilon_{\tilde{N}}$, there exists a neural network such that $|\varepsilon| \le \varepsilon_{\tilde{N}}$.

The basis functions that form the online network are $\left[1 \quad \sin(\hat{X}_i) \quad \sin(2\hat{X}_i) \quad \cdots \quad \sin(20\hat{X}_i)\right]$ for all i . Fourier series is used because of its boundedness, orthogonality and good non-linear function approximation capabilities. Twenty terms of the Fourier series are found to be sufficient. More terms could be included in a problem if needed. For this neural network architecture, the weight update rule is presented next.

5.4 *Weight Update Rule*

Let $\hat{U}_{opt}$ denote the control generated by the adaptive critic network controller for the nominal system

$$\dot{\hat{X}} = \hat{F}(\hat{X}) + B\hat{U} \quad (25)$$

Let $\hat{U}_{ex}$ denote the extra control being applied to compensate for uncertainty in the model (22). The total control applied is $\hat{U} = \hat{U}_{opt} + \hat{U}_{ex}$

Substituting the above expression for $\hat{U}$ in (22),

$$\dot{\hat{X}} = \hat{F}(\hat{X}) + B\left(\hat{U}_{opt} + \hat{U}_{ex}\right) + \hat{D}(\hat{X}) \quad (26)$$

By choosing $\hat{U}_{ex}$ as $-B^{-1}W'\phi(\hat{X})$, the uncertainty can be compensated for if the neural network is able to capture it. Substituting for $\hat{U}_{ex}$ and $\hat{D}(\hat{X})$ from (24),

$$\dot{\hat{X}} = \hat{F}(\hat{X}) + B\hat{U}_{opt} - W'\phi(\hat{X}) + W_{ideal}'\phi(\hat{X}) + \varepsilon \quad (27)$$

Now, an 'approximate' system is defined as follows

$$\dot{\hat{X}}_a = \hat{F}(\hat{X}) + B\hat{U}_{opt} + K\left(\hat{X} - \hat{X}_a\right) \quad (28)$$

where K is of the form $K = kI$ (k is scalar and I is the identity matrix).

The system described by (28) is an approximate system for (25). The extra term $K\left(\hat{X} - \hat{X}_a\right)$ is introduced to reduce the error between its states and the states of (27) and when $\hat{X} \rightarrow \hat{X}_a$, (25) and (28) become identical. By choosing the weight update rule as,

$$\dot{W} = \Gamma^{-1}\left(\phi(\hat{X})\hat{e}' - k\delta|\hat{e}|\phi(\hat{X})\phi(\hat{X})'W\right) \tag{29}$$

where $\hat{e} \triangleq \hat{X} - \hat{X}_a$

The weights are guaranteed to be bounded and the bound on error between the approximate plant and the actual plant can be made as small as possible by choosing design parameters k and δ. Γ is the learning rate of the network. The first term in (29) is used for approximating the uncertainty in the model and the second term ensures the boundedness of the weights. Proofs and discussion on approximation error of the network are provided in a previous work of the authors [Yadav].

5.5 *Implementation of the Robust/Optimal Control*

Figure 2 shows the control solution implementation scheme.

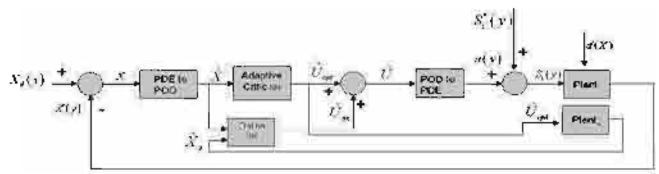

Figure 2. Implementation of Extra Control for Robustness

6. Application to Car Problem

It can be seen from (3) that the coefficients in the model are functions of the desired temperature profile. Control scheme was presented for the profile given by X_d. For any other desired profile (say X_d^*), the equation for deviation of states X from X_d^* can be written in a form similar to (7) as,

$$\frac{\partial x}{\partial t} = \alpha_1\left(\frac{\partial^2 x}{\partial y^2}\right) + f\left(x, X_d^*\right) + u \tag{30}$$

where $x = X - X_d^*$

It can be seen that the optimal control scheme developed for (7) cannot be applied to achieve X_d^*. To achieve any desired profile, the adaptive critic network must be trained again. In order to use the same network trained for X_d to achieve X_d^*, add and subtract $f\left(x, X_d\right)$ in (7) to get (30) in a form similar to (22) as

$$\frac{\partial x}{\partial t} = \alpha_1\left(\frac{\partial^2 x}{\partial y^2}\right) + f\left(x, X_d\right) + u + D(x) \tag{31}$$

where $D(x) = \left(f\left(x, X_d^*\right) - f\left(x, X_d\right)\right)$

By using the online neural network approach to compensate for this unmodeled dynamics, *any desired profile* X_d^* can be achieved.

7. Numerical Results

Numerical simulations were performed in MATLAB. Five basis functions were found to be sufficient to adequately describe the thermal model in finite-dimensions. Values of the parameters used in the simulations are presented in the table below.

Parameter	Definition	Value
q	Cost due to deviation from desired state	1
r	Cost due to control	1
k	Design parameter for online network	5
δ	Design parameter for online network	.1
Γ	Learning rate of online network	$.75I_5$

7.1 *SNAC Controller*

This section presents the results obtained by using the SNAC controller to achieve a constant temperature of $120°C$ across the plate. It can be seen from the three-dimensional temperature history presented in Figure 3 that the desired final temperature profile is reached. Also the control effort from the adaptive critic controller (POD side) transformed to the original control effort can be seen in figure 3. The total control to be applied is the sum of network output and steady state control. Note that the basis functions are used to transform this control effort to a true control and applied to the original model.

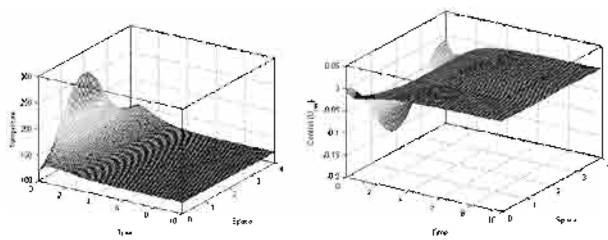

Figure 3. Temperature and Control History resp.

The method proposed in section 6 is used to achieve an exponential profile which is a complex task. The parameters used in the synthesis of the online network are $k = 5$ and $\delta = .01$. Figure 4 shows the desired temperature profile and the actual temperature at steady state. Note that they are identical.

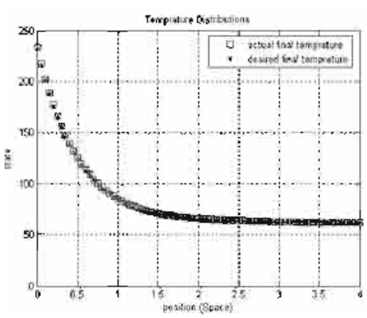

Figure 4. Temperature Distribution

7.2 *Online Neural Network for modeling uncertainties.*

In this section, results are presented for the case when radiation effects in the system are ignored in the preliminary controller design. An online neural network is then used to compensate for the modeling errors. Without the online network, the controller is not able to compensate for the unmodeled dynamics. The final achieved temperature and desired temperature are shown in figure 5. Figure 6 presents the actual modeling uncertainty and its network approximation. It can be seen from figure 7 that the error between modeling uncertainty and actual uncertainty goes to zero.

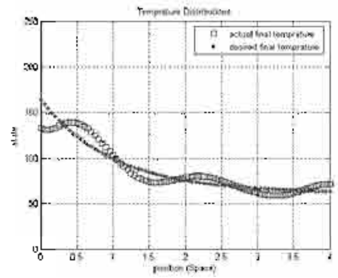

Figure 5. Temperature distribution

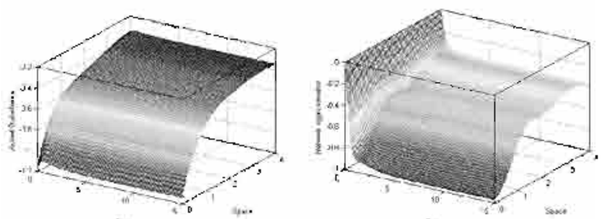

Figure 6. Actual uncertainty and modeling uncertainty

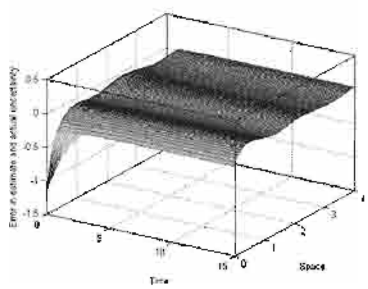

Figure 7. Error Between modeling uncertainty and actual uncertainty

It can be seen from figure 6 that there is an error between the desired and actual temperature profiles. This error is due to the approximation error of POD functions. This can be reduced by using more number of basis functions (figure. 8).

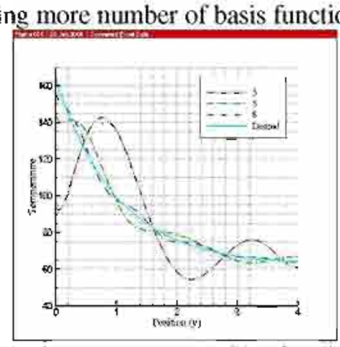

Figure 8. Final temperature profiles for different basis functions

7.3 Online Neural Network for parametric uncertainties.

This section presents the simulation results where the controller design is shown to be robust to parametric uncertainties. It is demonstrated by initially assuming an incorrect value of emissivity of the material in the initial controller design. The actual value is .8 times the value used for control synthesis in SNAC. The online neural network proposed is used to compensate for the errors introduced due to this parametiric uncertainty. Figure 9 shows that the final temperature achieved with the online network and without it.

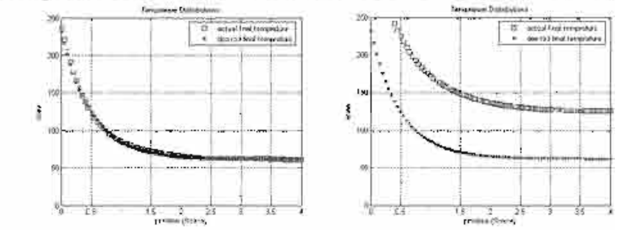

Figure 9. Final temperatures with and without uncertainty compensation.

CONCLUSIONS

In this study, approximate dynamic programming based formulations were used to synthesize suboptimal neurocontrollers for a cooling fin in a car's electronic module. An adaptive critic based SNAC controller was used to drive any given temperature profile to any desired profile. To compensate for the effect of uncertainty and to use the same adaptive critic network for achieving any desired profile, an online neural network was used. The weight update rule proposed in this paper ensures boundedness of the weight estimates and hence relaxes the PE condition. It was shown in simulation studies that the proposed robust control scheme can achieve any desired end profile and can compensate for modeling errors and parametric uncertainties.

REFERENCES

[1] Bryson A. E. and Ho Y. C., Applied Optimal Control, London: Taylor and Francis. 1975

[2] Curtain R. F. and Zwart H. J., An Introduction to Infinite Dimensional Linear Systems Theory, Springer-Verlag, New York, 1995.

[3] Holmes P., Lumley J. L. and Berkooz G., Turbulence, Coherent Structures, Dynamical Systems and Symmetry, Cambridge University Press, 1996, pp. 87-154.

[4] Miller A. F., Basic Heat and Mass Transfer, Richard D. Irwin Inc., Concord, MA, 1995.

[5] Padhi, R., Unnikrishnan N. and Balakrishnan. S. N., Optimal Control Synthesis of a Class of Nonlinear Systems Using Single Network Adaptive Critics, Submitted to ACC, 2004.

[6] aRavindran S. S., Proper Orthogonal Decomposition in Optimal Control of Fluids, NASA/TM-1999-209113.

[7] Slotine J-J. E. and Li W., Applied Nonlinear Control, Prentice Hall Inc, New Jersey, 1991.

[8] Yadav V, Padhi R, Balakrishnan S.N. Robust/Optimal Temperature Profile Control of a Re-Entry Vehicle Using Neural Networks. AIAA AFM, Keystone, Colorado, 21-24 Aug 2006